

SOME NEW DISCRETE FRACTIONAL INEQUALITIES AND THEIR APPLICATIONS IN FRACTIONAL DIFFERENCE EQUATIONS

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Abstract. In this paper, some new Gronwall-Bellman type discrete fractional difference inequalities and fractional sum inequalities are established. Based on the theory of discrete fractional calculus, explicit bounds for unknown functions concerned are presented. These inequalities can be used as a handy tool in the qualitative analysis of solutions of discrete fractional difference equations. As for applications, we apply the presented results to research boundedness, uniqueness, and continuous dependence on the initial value for the solutions of certain initial value problems of fractional difference equations.

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