

REFINEMENTS OF BOUNDS FOR NEUMAN MEANS IN TERMS OF ARITHMETIC AND CONTRAHARMONIC MEANS

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Abstract. In this paper, we present the sharp upper and lower bounds for the Neuman means S_{AC} and S_{CA} in terms of the arithmetic mean A and contraharmonic mean C . The given results are the improvements of some known results.

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