

THE HORNICH–HLAWKA INEQUALITY AND BERNSTEIN FUNCTIONS

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Abstract. The Hornich-Hlawka inequality for three real numbers is extended from the identity function to all Bernstein functions on the half-line. For vectors in a Euclidean space it is shown to hold for the square-root function.

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