

ON SOME INEQUALITIES FOR THE IDENTRIC, LOGARITHMIC AND RELATED MEANS

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Abstract. We offer new proofs, refinements as well as new results related to classical means of two variables, including the identric and logarithmic means.

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