

THE ISOPERIMETRIC INEQUALITY AND ITS STABILITY

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Abstract. In this paper, we deals with the isoperimetric-type inequalities for the closed convex curve in the Euclidean plane \mathbb{R}^2 . In fact we establish a family of parametric inequalities involving the following geometric functionals associated to the given closed convex curve with a simple Fourier series proof: length of the curve, areas of the region included by the curve and the locus of curvature centers, and integral of the curvature radii of the curve and the locus of curvature centers. Using our isoperimetric-type inequalities, we also derive some new geometric Bonnesen-type inequalities. Furthermore, we investigate the stability property of such inequalities (near equality implies curve nearly circular).

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