

## ON THE STABILITY OF METRIC SEMIGROUP HOMOMORPHISMS

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*Abstract.* In this paper, we investigate the stability of the homomorphism equation  $f(x \circ_1 y) = f(x) \circ_2 f(y)$  between semigroups  $(G_1, \circ_1)$  and  $(G_2, \circ_2)$ , where the binary operation  $\circ_i$  is square-symmetric on the set  $G_i$  for  $i = 1, 2$ . Our results generalize the classical theorem of Hyers concerning the stability of the Cauchy additive equation.

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