AN OPERATOR $\alpha$–GEOMETRIC MEAN INEQUALITY

XIAOHUI FU

Abstract. We square operator $\alpha$-geometric mean inequality as follows: If $0 < m_1 \leq A \leq M_1$ and $0 < m_2 \leq B \leq M_2$ for some positive real numbers $m_1 < M_1$ and $m_2 < M_2$, then for every unital positive linear map $\Phi$ and $\alpha \in [0, 1]$, the following inequality holds:

$$(\Phi(A))^{\alpha} \Phi(B) \leq \left( \frac{(M_1 + m_1)^2((M_1 + m_1)^{-1}(M_2 + m_2))^{2\alpha}}{4(m_2M_2)^\alpha(m_1M_1)^{1-\alpha}} \right)^2 \Phi^2(A)^{\alpha}B).$$


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REFERENCES