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# AN OPERATOR $\alpha$–GEOMETRIC MEAN INEQUALITY

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*Abstract.* We square operator $\alpha$-geometric mean inequality as follows: If $0 < m_1 \leqslant A \leqslant M_1$ and $0 < m_2 \leqslant B \leqslant M_2$ for some positive real numbers $m_1 < M_1$ and $m_2 < M_2$, then for every unital positive linear map $\Phi$ and $\alpha \in [0,1]$, the following inequality holds:

$$(\Phi(A)\sharp_\alpha\Phi(B))^2 \leqslant \left(\frac{(M_1+m_1)^2((M_1+m_1)^{-1}(M_2+m_2))^{2\alpha}}{4(m_2M_2)^\alpha(m_1M_1)^{(1-\alpha)}}\right)^2 \Phi^2(A\sharp_\alpha B).$$

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## REFERENCES

[1] R. BHATIA, F. KITTANEH, *Notes on matrix arithmetic-geometric mean inequalities*, Linear Algebra Appl. **308** (2000), 203–211.
[2] T. FURUTA, J. MIĆIĆ, J. E. PEČARIĆ AND Y. SEO, *Mond-Pečarić Method in Operator Inequalities*, Monographs in inequalities 1, Element, Zagreb, 2005.
[3] K. KUBO, T. ANDO, *Means of positive linear operators*, Math. Ann., **246** (1980), 205–224.
[4] M. LIN, *Squaring a reverse AM-GM inequality*, Studia Math. **215** (2013), 187–194.
[5] M. LIN, *On an operator Kantorovich inequality for positive linear maps*, J. Math. Anal. Appl. **402** (2013), 127–132.
[6] A. W. MARSHALL, I. OLKIN, *Matrix versions of Cauchy and Kantorovich inequalities*, Aequationes Math. **40** (1990), 89–93.
[7] J. PEČARIĆ, T. FURUTA, J. MIĆIĆ HOT AND Y. SEO, *Mond Pečarić method in operator inequalities*, Element, Zagreb, 2005.
[8] Y. SEO, *Reverses of Ando's inequality for positive linear maps*, Math. Inequal. Appl. **14** (2011), 905–910.