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ON BOUNDARY DOMINATION IN THE JENSEN–MERCER INEQUALITY

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Abstract. The main purpose of this, mainly expository, paper is to give various arguments that the boundary domination is a crucial property for the Jensen-Mercer inequality. Although this is an obvious property of convex functions and it is already expressed in the Jensen inequality it seems that the Jensen-Mercer inequality contains this information in a more vivid, explicit sense. This domination is presented using Steffensen-Popoviciu measures, The Majorization Theorem and a crude domination of weights of vertices in the multidimensional case (polytopes, simplices).

Mathematics subject classification (2010): 26A51, 26D15.

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