ON SEIFFERT–LIKE MEANS

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Abstract. We investigate the representation of homogeneous, symmetric means in the form

$$M(x,y) = \frac{x-y}{2f\left(\frac{x-y}{x+y}\right)}.$$

This allows for a new approach to comparing means. As an example, we provide optimal estimate of the form

$$(1-\mu)\min(x,y) + \mu\max(x,y) \leqslant M(x,y) \leqslant (1-\nu)\min(x,y) + \nu\max(x,y)$$

and

$$M\left(\tfrac{x+y}{2} - \mu\tfrac{x-y}{2}, \tfrac{x+y}{2} + \mu\tfrac{x-y}{2}\right) \leqslant N(x,y) \leqslant M\left(\tfrac{x+y}{2} - \nu\tfrac{x-y}{2}, \tfrac{x+y}{2} + \nu\tfrac{x-y}{2}\right)$$

for some known means.
We also introduce an integral operator on the set of means and investigate its properties.

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