ON SEIFFERT–LIKE MEANS

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Abstract. We investigate the representation of homogeneous, symmetric means in the form

\[ M(x, y) = \frac{x - y}{2f\left(\frac{x - y}{x + y}\right)} \]

This allows for a new approach to comparing means. As an example, we provide optimal estimate of the form

\[ (1 - \mu)\min(x, y) + \mu\max(x, y) \leq M(x, y) \leq (1 - \nu)\min(x, y) + \nu\max(x, y) \]

and

\[ M\left(\frac{x + y}{2} - \mu\frac{x - y}{2}, \frac{x + y}{2} + \mu\frac{x - y}{2}\right) \leq N(x, y) \leq M\left(\frac{x + y}{2} - \nu\frac{x - y}{2}, \frac{x + y}{2} + \nu\frac{x - y}{2}\right) \]

for some known means.

We also introduce an integral operator on the set of means and investigate its properties.


Keywords and phrases: Seiffert mean, logarithmic mean, Seiffert function.

REFERENCES


