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ASYMPTOTIC EXPANSION OF THE ARITHMETIC–GEOMETRIC MEAN AND RELATED INEQUALITIES

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Abstract. Asymptotic expansion of the arithmetic-geometric mean is obtained and it is used to analyze inequalities and relations between arithmetic-geometric mean and other classical means.

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