

ON THE SUM OF SQUARED LOGARITHMS INEQUALITY AND RELATED INEQUALITIES

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Abstract. We consider the sum of squared logarithms inequality and investigate possible connections with the theory of majorization. We also discuss alternative sufficient conditions on two sets of vectors $a, b \in \mathbb{R}_+^n$ so that

$$\sum_{i=1}^n (\log a_i)^2 \leq \sum_{i=1}^n (\log b_i)^2.$$

Generalizations of some inequalities from information theory are obtained, including a generalized information inequality and a generalized log sum inequality, which states for $a, b \in \mathbb{R}_+^n$ and $k_1, \dots, k_n \in [0, \infty)$:

$$\sum_{i=1}^n a_i \log \prod_{s=1}^m \left(\frac{a_i}{b_i} + k_s \right) \geq \log \prod_{s=1}^m (1 + k_s).$$

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