

ESTIMATIONS OF HERON MEANS FOR POSITIVE OPERATORS

MASATOSHI FUJII, SHIGERU FURUICHI AND RITSUO NAKAMOTO

Abstract. The arithmetic-geometric mean inequality induces the path of Heron means through these two means by $H_r^\mu(A, B) = r(A\sharp_\mu B) + (1-r)(A\nabla_\mu B)$ for each $\mu \in [0, 1]$, $r \in \mathbb{R}$ and positive operators A, B on a Hilbert space. In this note, we estimate $H_r^\mu(A, B)$ by the harmonic mean. As an application of this method, we refine the arithmetic-geometric mean inequality under the assumption of the strict order $A - B \succcurlyeq m > 0$.

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