Journal of Mathematical Inequalities
Volume 10, Number 1 (2016), 123–133
doi:10.7153/jmi-10-10

CONTINUOUS MONOTONE MAPS ON MATRICES FOR ORDERS INDUCED BY THE GROUP INVERSE

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Abstract. We characterize continuous injective maps on the set of complex matrices which are monotone with respect to $\overset{\sharp}{\leqslant}$-order or $\overset{cn}{\leqslant}$-order. In particular, we prove that all such maps must be automatically $\mathbb{R}$-linear and surjective. We also present several examples of monotone maps showing that our assumptions are indispensable.

Mathematics subject classification (2010): 15A04.
Keywords and phrases: Monotone matrix maps, group inverse, matrix partial orders.

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