

## IMPROVED CAUCHY–SCHWARZ NORM INEQUALITY FOR OPERATORS

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*Abstract.* Let  $A$ ,  $B$  and  $X$  be operators on a complex separable Hilbert space such that  $A$  and  $B$  are positive. Cauchy-Schwarz norm inequality for operators asserts that

$$\left\| \left\| A^{\frac{1}{2}} X B^{\frac{1}{2}} \right\|^r \right\|^2 \leq \| |AX|^r \| \cdot \| |XB|^r \|,$$

for any real number  $r > 0$  and every unitarily invariant norm  $\|\cdot\|$ . In this article we derive several refinements of Cauchy-Schwarz norm inequality for operators. In particular, we show improvements for the results of Hiai and Zhan [Linear Algebra Appl. 341 (2002) 151–169]. Besides, new type inequalities close to Cauchy-Schwarz norm inequality will be introduced.

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