

NONLINEAR RETARDED INTEGRAL INEQUALITIES OF GRONWALL–BELLMAN TYPE AND APPLICATIONS

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Abstract. The aim of this paper is to establish some new nonlinear retarded integral inequalities of Gronwall–Bellman type. More accurately we extend certain results which have been proved in El-Owaidy et. al, [12] and Abdeldaim and Yakout [2] to some nonlinear retarded integral inequalities. We also give some applications to estimate the solutions of some nonlinear retarded differential equations to illustrate the effectiveness of some from our results.

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REFERENCES

- [1] A. ABDELDAIM, A. A. EL-DEEB, *Some new retarded nonlinear integral inequalities with iterated integrals and their applications in retarded differential equations and integral equations*, Journal of Fractional Calculus and Applications. Vol. **5** (3S) No. 9 (2014), pp. 1–9.
- [2] A. ABDELDAIM, M. YAKOUT, *On some new integral inequalities of Gronwall–Bellman–Pachpatte type*, Appl Math Comput. **217** (2011), 7887–7899.
- [3] A. ABDELDAIM, A. A. EL-DEEB, *On some new nonlinear retarded integral inequalities with iterated integrals and their applications in integro-differential equations*, Br. J. Math. Comput. Sci. **5** (2015), 479–491. Article No. BJMCS.2015.034.
- [4] R. P. AGARWAL, S. DENG, W. ZHANG, *Generalization of a retarded Gronwall-like inequality and its applications*, Appl Math Comput. **165** (2005), 599–612.
- [5] R. P. AGARWAL, Y. H. KIM, S. K. SEN, *New retarded integral inequalities with applications*, J. Inequal. Appl. (2008), Article ID 908784(2008).
- [6] R. BELLMAN, *The stability of solutions of linear differential equations*, Duke Math J. **10** (1943), 643–647.
- [7] D. BIANOVE AND P. SIMEOVE, *Integral Inequalities and Applications*, New York, Kluwer Academic Publishers, (1992).
- [8] I. A. BIHARI, *A generalization of a lemma of Bellman and its application to uniqueness problem of differential equation*, Acta Math Acad Sci Hung. **7** (1956), 81–94.
- [9] W. S. CHEUNG, *Some new nonlinear inequalities and applications to boundary value problems*, Nonlinear Anal. **64** (2006), 2112–2128.
- [10] H. EL-OWAIDY, A. ABDELDAIM, A. A. EL-DEEB, *On some new nonlinear retarded integral inequalities with iterated integrals and their applications in differential-integral equations*, Mathematical Sciences Letters journal, **3**, No. 3 (2014), 157–164.
- [11] H. EL-OWAIDY, A. A. RAGAB, AND A. ABDELDAIM, *On some integral inequalities of Gronwall–Bellman type*, Appl. Math Comput., **106** (1999), 289–303.
- [12] H. EL-OWAIDY, A. A. RAGAB, W. ABUELELA, A. A. EL-DEEB, *On some new nonlinear integral inequalities of Gronwall–Bellman type*, KYUNGPOOK Math. J. **54** (2014), 555–575.
- [13] T. H. GRONWALL, *Note on the derivatives with respect to a parameter of the solutions of a system of differential equations*, Ann Math. **20** (1919), 292–296.
- [14] Y. H. KIM, *On some new integral inequalities for functions in one and two variables*, Acta Math Sinica. **21** (2005), 423–434.

- [15] O. LIPOVAN, *A retarded Gronwall-like inequality and its applications*, J. Math Anal Appl. **252** (2000), 389–401.
- [16] B. G. PACHPATTE, *Inequalities for Differential and Integral Equations*, Academic Press London (1998).
- [17] B. G. PACHPATTE, *Explicit bound on certain integral inequalities*, J. Math. analysis and Applications, **267** (2002), 48–61.
- [18] M. H. M. RASHID, *Explicit bound on retarded Gronwall-Bellman inequality*, Tamkang J. Math. **43** (2012), 99–108.
- [19] W. S. WANG, *A generalized retarded Gronwall-like inequality in two variables and applications to BVP*, Appl Math Comput. **191** (2007), 144–154.
- [20] W. S. WANG, C. SHEN, *On a generalized retarded integral inequality with two variables*, J. Inequal. Appl. (2008), Article ID 518646 (2008).
- [21] W. S. WANG, Z. LI, Y. LI, Y. HUANG, *Nonlinear retarded integral inequalities with two variables and applications*, J. Inequal. Appl. (2010), Article ID 240790 (2010).
- [22] W. S. WANG, R. C. LUO, Z. LI, *A new nonlinear retarded integral inequality and its application*, J. Inequal. Appl. (2010), Article ID 462163 (2010).
- [23] W. S. WANG, *Some generalized nonlinear retarded integral inequalities with applications*, J. Inequal. Appl. (2012), 2012:31.
- [24] W. S. WANG, *Some New generalized retarded nonlinear integral inequalities with iterated integrals and thier applications*, J. Inequal. Appl. (2012):236.
- [25] A. KHAN ZAREEN, *Integral inequalities of Gronwall-Bellman Type*, Applied Mathematics. **5** (2015), 3484–3488.