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SHARP BOUNDS FOR NEUMAN MEANS IN TERMS OF GEOMETRIC, ARITHMETIC AND QUADRATIC MEANS

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Abstract. In this paper, we find the greatest values α_1, α_2, α_3, α_4, α_5, α_6, α_7, α_8 and the least values β_1, β_2, β_3, β_4, β_5, β_6, β_7, β_8 such that the double inequalities

$$A^{\alpha_1}(a,b)G^{1-\alpha_1}(a,b) < N_{GA}(a,b) < A^{\beta_1}(a,b)G^{1-\beta_1}(a,b),$$

$$\frac{\alpha_2}{G(a,b)} + \frac{1-\alpha_2}{A(a,b)} < \frac{1}{N_{GA}(a,b)} < \frac{\beta_2}{G(a,b)} + \frac{1-\beta_2}{A(a,b)},$$

$$A^{\alpha_3}(a,b)G^{1-\alpha_3}(a,b) < N_{AG}(a,b) < A^{\beta_3}(a,b)G^{1-\beta_3}(a,b),$$

$$\frac{\alpha_4}{G(a,b)} + \frac{1-\alpha_4}{A(a,b)} < \frac{1}{N_{AG}(a,b)} < \frac{\beta_4}{G(a,b)} + \frac{1-\beta_4}{A(a,b)},$$

$$Q^{\alpha_5}(a,b)A^{1-\alpha_5}(a,b) < N_{AQ}(a,b) < Q^{\beta_5}(a,b)A^{1-\beta_5}(a,b),$$

$$\frac{\alpha_6}{A(a,b)} + \frac{1-\alpha_6}{Q(a,b)} < \frac{1}{N_{AQ}(a,b)} < \frac{\beta_6}{A(a,b)} + \frac{1-\beta_6}{Q(a,b)},$$

$$Q^{\alpha_7}(a,b)A^{1-\alpha_7}(a,b) < N_{QA}(a,b) < Q^{\beta_7}(a,b)A^{1-\beta_7}(a,b),$$

$$\frac{\alpha_8}{A(a,b)} + \frac{1-\alpha_8}{Q(a,b)} < \frac{1}{N_{QA}(a,b)} < \frac{\beta_8}{A(a,b)} + \frac{1-\beta_8}{Q(a,b)}$$

hold for all $a,b>0$ with $a \neq b$, where G, A and Q are respectively the geometric, arithmetic and quadratic means, and N_{GA}, N_{AG}, N_{AQ} and N_{QA} are the Neuman means derived from the Schwab-Borchardt mean.

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