

PARAMETRIC MARCINKIEWICZ INTEGRALS ON THE WEIGHTED HARDY AND WEAK HARDY SPACES

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Abstract. Let $0 < \rho < n$ and μ_{Ω}^{ρ} be the parametric Marcinkiewicz integral. In this paper, by using the atomic decomposition theory of weighted Hardy and weak Hardy spaces, we will obtain the boundedness properties of μ_{Ω}^{ρ} on these spaces, under the Lipschitz condition imposed on the kernel Ω .

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