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A SHARP DOUBLE INEQUALITY INVOLVING TRIGONOMETRIC FUNCTIONS AND ITS APPLICATIONS

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Abstract. We present the best possible parameters $p,q \in (0,1]$ such that the double inequality $\frac{1}{3p^2}\cos(px)+1-\frac{1}{3p^2} < \frac{\sin(x)}{x} < \frac{1}{3q^2}\cos(qx)+1-\frac{1}{3q^2}$ holds for all $x \in (0,\pi/2)$. As applications, some new inequalities for the sine integral, Catalan constant and Schwab-Borchardt mean are found.

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