

INEQUALITIES WITH CURVATURE AND THEIR STABILITY ESTIMATES FOR CONVEX CURVES

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Abstract. In this paper, we deal with a geometric inequality for a closed convex plane curve γ involving the area, the perimeter, the curvature of γ . Together with an inequality obtained by Lin and Tsai in [12], we conclude the upper and lower bound estimates for the integration of the squared curvature radius of γ and show the stability results of the corresponding inequalities.

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