ON THE NORMS OF r-CIRCULANT MATRICES
WITH THE HYPERHARMONIC NUMBERS

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Abstract. In this paper, we study norms of circulant matrices $H = \mathrm{Circ}(H_0^{(k)}, \ H_1^{(k)}, \ldots, H_{n-1}^{(k)})$, $\widehat{H} = \mathrm{Circ}(H_k^{(0)}, \ H_k^{(1)}, \ldots, H_k^{(n-1)})$ and $r-$ circulant matrices $H_r = \mathrm{Circr}(H_0^{(k)}, H_1^{(k)}, \ldots, H_{n-1}^{(k)})$, $\widehat{H_r} = \mathrm{Circr}(H_k^{(0)}, H_k^{(1)}, \ldots, H_k^{(n-1)})$, where $H_n^{(k)}$ denotes the nth hyperharmonic number of order r.

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