

## SOME MEASURE THEORETIC ASPECTS OF STEFFENSEN'S AND REVERSED STEFFENSEN'S INEQUALITY

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*Abstract.* We find necessary and sufficient conditions for Steffensen's and inverse Steffensen's inequality in measure theoretic settings. We also explain natural transition from the Riemann integral in Steffensen's inequality to an integral with measure.

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