

SOME BENNETT–COPSON TYPE INEQUALITIES ON TIME SCALES

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Abstract. In this paper, we will prove some new dynamic inequalities with two different weighted functions on a time scale. As special cases, the inequalities contain some dynamic inequalities on time scales and also involve some discrete inequalities formulated by Copson, Leindler, Bennett, Chen and Yang. The results will be proved by using Hölder's inequality and Minkowski's inequality on time scales.

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