ON PÓLYA–SZEGÖ AND CHEBYSHEV TYPES INEQUALITIES INVOLVING THE RIEMANN–LIOUVILLE FRACTIONAL INTEGRAL OPERATORS

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Abstract. In this paper, we investigate some new Pólya-Szegö type integral inequalities involving the Riemann-Liouville fractional integral operator, and use them to prove some fractional integral inequalities of Chebyshev type, concerning the integral of the product of two functions and the product of two integrals. Certain special cases are also considered. Finally, examples for constructing the bounding functions are also given.

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