

## THE LOG-CONVEXITY OF GENOCCHI NUMBERS AND THE MONOTONICITY OF SOME SEQUENCES RELATED TO GENOCCHI NUMBERS

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*Abstract.* In this paper, we investigate the properties of Genocchi number  $\{G_n\}_{n \geq 1}$ . We prove that the sequence  $\{|G_{2n}|\}_{n \geq 1}$  is log-convex. In addition, we discuss the monotonicity of some sequences related to  $\{G_n\}_{n \geq 1}$ . In particular, we show that  $\{\sqrt[n]{|G_{2n}|}\}_{n \geq 1}$  is strictly increasing and  $\{\sqrt[n+1]{|G_{2n+2}|} / \sqrt[n]{|G_{2n}|}\}_{n \geq 2}$  is strictly decreasing.

*Mathematics subject classification (2010):* 05A20, 05A10, 05A16, 11B68, 11B83.

*Keywords and phrases:* Genocchi numbers, Bernoulli numbers, tangent numbers, log-convexity, monotonicity.

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