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SQUARING OPERATOR α–GEOMETRIC MEAN INEQUALITY

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Abstract. In this paper, we square operator α-geometric mean inequality as follows: If $0 < m_1^2 \leqslant A \leqslant M_1^2$ and $0 < m_2^2 \leqslant B \leqslant M_2^2$ for some positive real numbers $m_1 < M_1$ and $m_2 < M_2$, then for every unital positive linear map Φ and $\alpha \in [0,1]$, the following inequality holds:

$$\{\Phi(A)\sharp_\alpha\Phi(B)\}^2 \leqslant \frac{K\left((\frac{m_2}{M_1})^2,(\frac{M_2}{m_1})^2,\alpha\right)^{-2}(G+g)^2}{4Gg}\Phi^2(A\sharp_\alpha B)$$

where the generalized Kantorovich constant $K\left((\frac{m_2}{M_1})^2,(\frac{M_2}{m_1})^2,\alpha\right)$ is defined by

$$K(m,M,\alpha) = \frac{mM^\alpha - Mm^\alpha}{(\alpha-1)(M-m)}\left(\frac{\alpha-1}{\alpha}\,\frac{M^\alpha - m^\alpha}{mM^\alpha - Mm^\alpha}\right)^\alpha$$

and $G = M_1(M_1^{-1}M_2)^{2\alpha}M_1$, $g = m_1(m_1^{-1}m_2)^{2\alpha}m_1$.

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