SHARP LEHRER MEAN BOUNDS FOR NEUMAN MEANS WITH APPLICATIONS

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Abstract. In the article, we present the best possible parameters \( \alpha_1, \alpha_2, \alpha_3, \alpha_4 \) and \( \beta_1, \beta_2, \beta_3, \beta_4 \) such that the double inequalities

\[
L_{\alpha_1}(a, b) < N_{AG}(a, b) < L_{\beta_1}(a, b), \quad L_{\alpha_2}(a, b) < N_{GA}(a, b) < L_{\beta_2}(a, b), \\
L_{\alpha_3}(a, b) < N_{QA}(a, b) < L_{\beta_3}(a, b), \quad L_{\alpha_4}(a, b) < N_{AQ}(a, b) < L_{\beta_4}(a, b)
\]

hold for all \( a, b > 0 \) with \( a \neq b \), where \( L_p(a, b) = \left( a^{p+1} + b^{p+1} \right) / (a^p + b^p) \) is the \( p \)th Lehmer mean, and \( N_{AG}(a, b), N_{GA}(a, b), N_{QA}(a, b) \) and \( N_{AQ}(a, b) \) are the Neuman means. As applications, we find several sharp inequalities involving the hyperbolic, trigonometric and inverse trigonometric functions.

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