

## SHARP LEHMER MEAN BOUNDS FOR NEUMAN MEANS WITH APPLICATIONS

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*Abstract.* In the article, we present the best possible parameters  $\alpha_1, \alpha_2, \alpha_3, \alpha_4$  and  $\beta_1, \beta_2, \beta_3, \beta_4$  such that the double inequalities

$$L_{\alpha_1}(a, b) < N_{AG}(a, b) < L_{\beta_1}(a, b), \quad L_{\alpha_2}(a, b) < N_{GA}(a, b) < L_{\beta_2}(a, b),$$

$$L_{\alpha_3}(a, b) < N_{QA}(a, b) < L_{\beta_3}(a, b), \quad L_{\alpha_4}(a, b) < N_{AQ}(a, b) < L_{\beta_4}(a, b)$$

hold for all  $a, b > 0$  with  $a \neq b$ , where  $L_p(a, b) = (a^{p+1} + b^{p+1}) / (a^p + b^p)$  is the  $p$ th Lehmer mean, and  $N_{AG}(a, b)$ ,  $N_{GA}(a, b)$ ,  $N_{QA}(a, b)$  and  $N_{AQ}(a, b)$  are the Neuman means. As applications, we find several sharp inequalities involving the hyperbolic, trigonometric and inverse trigonometric functions.

*Mathematics subject classification (2010):* 26E60, 26D05, 26D07.

*Keywords and phrases:* Neuman mean, Lehmer mean, Schwab-Borchardt mean, geometric mean, arithmetic mean, quadratic mean, hyperbolic function, trigonometric function, inverse trigonometric function.

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