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WEIGHTED KERNEL OPERATORS IN $L^{p(x)}(\mathbb{R}_+)$ SPACES

ALEXANDER MESKHI AND MUHAMMAD ASAD ZAIGHUM

Abstract. Necessary and sufficient conditions on a weight v governing the boundedness/ compactness of the weighted kernel operator $K_v f(x) = v(x) \int_0^x k(x,t) f(t) dt$ from the variable exponent Lebesgue spaces $L^{p(\cdot)}(\mathbb{R}_+)$ into another one $L^{q(\cdot)}(\mathbb{R}_+)$ is established under the local log–Hölder continuity condition and the decay condition at infinity on exponents. The distance between K_v and the class of compact integral operators acting from $L^{p(\cdot)}(\mathbb{R}_+)$ to $L^{q(\cdot)}(\mathbb{R}_+)$ (measure of non–compactness) is also estimated from above and below.

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