

A GEOMETRIC INEQUALITY WITH APPLICATIONS

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Abstract. In this paper, we present a new geometric inequality which involves an arbitrary point in the plane of a triangle. A simpler proof of a known inequality with one parameter is obtained by using our result. We also derive the famous Sondat fundamental triangle inequality from it.

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