

## WEIGHTED NORM INEQUALITIES FOR TOEPLITZ TYPE OPERATOR ASSOCIATED TO SINGULAR INTEGRAL OPERATOR WITH NON-SMOOTH KERNEL

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*Abstract.* Let  $T^{k,1}$  be singular integrals with non-smooth kernels, which are associated with an approximation to the identity or  $\pm I$  (the identity operator),  $T^{k,2}$  and  $T^{k,4}$  are the linear operators,  $T^{k,3} = \pm I$ . Denote the Toeplitz type operator by

$$T_b = \sum_{k=1}^m (T^{k,1} M_b I_\alpha T^{k,2} + T^{k,3} I_\alpha M_b T^{k,4}),$$

where  $M_b f(x) = b(x)f(x)$ , and  $I_\alpha$  is the fractional integral operator. In this paper, we establish the sharp maximal function estimates for  $T_b$  when  $b$  belongs to weighted Lipschitz function space. As an application, the boundedness of the operator on weighted Lebesgue space is obtained.

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