

SCHUR CONVEXITY OF STOLARSKY'S EXTENDED MEAN VALUES

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Abstract. In the recent years, the Schur convexity and Schur geometrically convexity of Stolarsky's mean values have grabbed the focus of many mathematicians and researchers. In this article, the Schur convexity of Stolarsky's extended type mean values are discussed.

Mathematics subject classification (2010): Primary 26D10, 26D15.

Keywords and phrases: Heron mean, harmonic mean, Schur concavity, convexity.

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