

SOME NEW HARDY-TYPE INEQUALITIES IN q -ANALYSIS

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Abstract. We derive necessary and sufficient conditions (of Muckenhoupt-Bradley type) for the validity of q -analogs of (r, p) -weighted Hardy-type inequalities for all possible positive values of the parameters r and p . We also point out some possibilities to further develop the theory of Hardy-type inequalities in this new direction.

Mathematics subject classification (2010): 26D10, 26D15, 33D05, 39A13.

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