INITIAL COEFFICIENT BOUND FOR m–FOLD SYMMETRIC BI–λ–CONVEX FUNCTIONS

S. SIVASUBRAMANIAN AND R. SIVAKUMAR

Abstract. Let the functions $f(z) = z + a_2 z^2 + \cdots$ and its inverse f^{-1} be analytic and univalent in the unit disk. Such class of functions are called bi-univalent and denoted by σ [9]. In an article, Pommerenke [10] remarked that, for an m-fold symmetric functions in the class $\mathscr{P}$, the well known lemma stated by Caratheodary for a one fold symmetric functions in $\mathscr{P}$ still holds good. Making use of this remark, we introduce two new subclasses of *bi-univalent* functions in which both f and $f^{-1} = g$ are m-fold symmetric analytic functions with $(1 - \lambda)\dfrac{z f'(z)}{f(z)} + \lambda \left(1 + \dfrac{z f''(z)}{f'(z)} \right)$ and $(1 - \lambda)\dfrac{w g'(w)}{g(w)} + \lambda \left(1 + \dfrac{w g''(w)}{g'(w)} \right)$ in $\mathscr{P}$ and obtain coefficient bounds for functions in this new classes.

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