SOME HARDY INEQUALITIES ON THE SPHERE

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Abstract. In this paper we establish some Hardy inequalities related to the geodesic distance on the sphere and compute the corresponding sharp constants. Moreover, some Rellich inequalities have been also established and the constants are also sharp.

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