

## A POINCARÉ-TYPE INEQUALITY ON THE EUCLIDEAN UNIT SPHERE

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*Abstract.* We consider the second variation for the volume of convex bodies associated with the  $L_p$  Minkowski-Firey combination and obtain a Poincaré-type inequality on the Euclidean unit sphere  $S^{m-1}$ .

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