SYMMETRIZED CONVEXITY AND
HERMITE–HADAMARD TYPE INEQUALITIES

S. S. DRAGOMIR

Abstract. In this paper we extend the Hermite-Hadamard inequality to the class of symmetrized convex functions. The corresponding version for h-convex functions is also investigated. Some examples of interest are provided as well.

Mathematics subject classification (2010): 26D15, 25D10.

Keywords and phrases: Convex functions, Hermite-Hadamard inequality, integral inequalities, h-convex functions.

REFERENCES

[1] M. ALOMARI AND M. DARUS, *The Hadamard's inequality for s-convex function*, Int. J. Math. Anal. (Ruse) **2** (2008), no. 13–16, 639–646.

[2] M. ALOMARI AND M. DARUS, *Hadamard-type inequalities for s-convex functions*, Int. Math. Forum **3** (2008), no. 37–40, 1965–1975.

[3] G. A. ANASTASSIOU, *Univariate Ostrowski inequalities*, revisited, Monatsh. Math., **135** (2002), no. 3, 175–189.

[4] N. S. BARNETT, P. CERONE, S. S. DRAGOMIR, M. R. PINHEIRO AND A. SOFO, *Ostrowski type inequalities for functions whose modulus of the derivatives are convex and applications*, Inequality Theory and Applications, Vol. 2 (Chinju/Masan, 2001), 19–32, Nova Sci. Publ., Hauppauge, NY, 2003. Preprint: RGMIA Res. Rep. Coll. **5** (2002), No. 2, Art. 1 [Online http://rgmia.org/papers/v5n2/Paperwapp2q.pdf].

[5] E. F. BECKENBACH, *Convex functions*, Bull. Amer. Math. Soc. **54** (1948), 439–460.

[6] M. BOMBARDELLI AND S. VAROŠANEC, *Properties of h-convex functions related to the Hermite-Hadamard-Fejér inequalities*, Comput. Math. Appl. **58** (2009), no. 9, 1869–1877.

[7] W. W. BRECKNER, *Stetigkeitsaussagen für eine Klasse verallgemeinerter konvexer Funktionen in topologischen linearen Räumen*, (German) Publ. Inst. Math. (Beograd) (N.S.) **23** (37) (1978), 13–20.

[8] W. W. BRECKNER AND G. ORBÁN, *Continuity properties of rationally s-convex mappings with values in an ordered topological linear space*, Universitatea "Babeş-Bolyai", Facultatea de Matematica, Cluj-Napoca, 1978. viii+92 pp.

[9] P. CERONE AND S. S. DRAGOMIR, *Midpoint-type rules from an inequalities point of view, Ed. G. A. Anastassiou*, Handbook of Analytic-Computational Methods in Applied Mathematics, CRC Press, New York, 135–200.

[10] P. CERONE AND S. S. DRAGOMIR, *New bounds for the three-point rule involving the Riemann-Stieltjes integrals*, in Advances in Statistics Combinatorics and Related Areas, C. Gulati, et al. (Eds.), World Science Publishing, 2002, 53–62.

[11] P. CERONE, S. S. DRAGOMIR AND J. ROUMELIOTIS, *Some Ostrowski type inequalities for n-time differentiable mappings and applications*, Demonstratio Mathematica, **32** (2) (1999), 697—712.

[12] G. CRISTESCU, *Hadamard type inequalities for convolution of h-convex functions*, Ann. Tiberiu Popoviciu Semin. Funct. Equ. Approx. Convexity **8** (2010), 3–11.

[13] S. S. DRAGOMIR, *Ostrowski's inequality for monotonous mappings and applications*, J. KSIAM, **3** (1) (1999), 127–135.

[14] S. S. DRAGOMIR, *The Ostrowski's integral inequality for Lipschitzian mappings and applications*, Comp. Math. Appl. **38** (1999), 33–37.

[15] S. S. DRAGOMIR, *On the Ostrowski's inequality for Riemann-Stieltjes integral*, Korean J. Appl. Math., **7** (2000), 477–485.

[16] S. S. DRAGOMIR, *On the Ostrowski's inequality for mappings of bounded variation and applications*, Math. Ineq. & Appl., **4** (1) (2001), 33–40.

[17] S. S. DRAGOMIR, *On the Ostrowski inequality for Riemann-Stieltjes integral $\int_a^b f(t)du(t)$ where f is of Hölder type and u is of bounded variation and applications*, J. KSIAM, **5** (1) (2001), 35–45.

[18] S. S. DRAGOMIR, *Ostrowski type inequalities for isotonic linear functionals*, J. Inequal. Pure & Appl. Math., **3** (5) (2002), Art. 68.

[19] S. S. DRAGOMIR, *An inequality improving the first Hermite-Hadamard inequality for convex functions defined on linear spaces and applications for semi-inner products*, J. Inequal. Pure Appl. Math. **3** (2002), no. 2, Article 31, 8 pp.

[20] S. S. DRAGOMIR, *An inequality improving the second Hermite-Hadamard inequality for convex functions defined on linear spaces and applications for semi-inner products*, J. Inequal. Pure Appl. Math. **3** (2002), no. 3, Article 35.

[21] S. S. DRAGOMIR, *An Ostrowski like inequality for convex functions and applications*, Revista Math. Complutense, **16** (2) (2003), 373–382.

[22] S. S. DRAGOMIR, *Operator Inequalities of Ostrowski and Trapezoidal Type*, Springer Briefs in Mathematics, Springer, New York, 2012.

[23] S. S. DRAGOMIR, *Inequalities of Hermite-Hadamard type for h-convex functions on linear spaces*, Proyecciones **34**, 4 (2015), 323–341. Preprint RGMIA Res. Rep. Coll. **16** (2013), Art. 72 [Online `http://rgmia.org/papers/v16/v16a72.pdf`].

[24] S. S. DRAGOMIR, P. CERONE, J. ROUMELIOTIS AND S. WANG, *A weighted version of Ostrowski inequality for mappings of Hölder type and applications in numerical analysis*, Bull. Math. Soc. Sci. Math. Romanie, **42** (90) (4) (1999), 301–314.

[25] S. S. DRAGOMIR AND S. FITZPATRICK, *The Hadamard inequalities for s-convex functions in the second sense*, Demonstratio Math. **32** (1999), no. 4, 687–696.

[26] S. S. DRAGOMIR AND S. FITZPATRICK, *The Jensen inequality for s-Breckner convex functions in linear spaces*, Demonstratio Math. **33** (2000), no. 1, 43–49.

[27] S. S. DRAGOMIR AND B. MOND, *On Hadamard's inequality for a class of functions of Godunova and Levin*, Indian J. Math. **39** (1997), no. 1, 1–9.

[28] S. S. DRAGOMIR AND C. E. M. PEARCE, *On Jensen's inequality for a class of functions of Godunova and Levin*, Period. Math. Hungar. **33** (1996), no. 2, 93–100.

[29] S. S. DRAGOMIR AND C. E. M. PEARCE, *Quasi-convex functions and Hadamard's inequality*, Bull. Austral. Math. Soc. **57** (1998), 377–385.

[30] S. S. DRAGOMIR, J. PEČARIĆ AND L. PERSSON, *Some inequalities of Hadamard type*, Soochow J. Math. **21** (1995), no. 3, 335–341.

[31] S. S. DRAGOMIR AND TH. M. RASSIAS (Eds), *Ostrowski Type Inequalities and Applications in Numerical Integration*, Kluwer Academic Publisher, 2002.

[32] S. S. DRAGOMIR AND S. WANG, *A new inequality of Ostrowski's type in L_1-norm and applications to some special means and to some numerical quadrature rules*, Tamkang J. of Math., **28** (1997), 239–244.

[33] S. S. DRAGOMIR AND S. WANG, *Applications of Ostrowski's inequality to the estimation of error bounds for some special means and some numerical quadrature rules*, Appl. Math. Lett., **11** (1998), 105–109.

[34] S. S. DRAGOMIR AND S. WANG, *A new inequality of Ostrowski's type in L_p-norm and applications to some special means and to some numerical quadrature rules*, Indian J. of Math., **40** (3) (1998), 245–304.

[35] A. EL FARISSI, *Simple proof and refeinment of Hermite-Hadamard inequality*, J. Math. Ineq. **4** (2010), No. 3, 365–369.

[36] ABDALLAH EL FARISSI, MAAMAR BENBACHIR, MERIEM DAHMANE, *An extension of the Hermite-Hadamard inequality for convex symmetrized functions*, Real Anal. Exchange **38**, 2 (2012/13), 467–474.

[37] L. FEJÉR, *Über die Fourierreihen, II*, Math. Naturwiss. Anz Ungar. Akad. Wiss. **24** (1906), 369–390, (in Hungarian).

[38] E. K. GODUNOVA AND V. I. LEVIN, *Inequalities for functions of a broad class that contains convex, monotone and some other forms of functions*, (Russian) Numerical mathematics and mathematical physics (Russian), 138–142, 166, Moskov. Gos. Ped. Inst., Moscow, 1985.

[39] H. HUDZIK AND L. MALIGRANDA, *Some remarks on s-convex functions*, Aequationes Math. **48** (1994), no. 1, 100–111.

[40] E. KIKIANTY AND S. S. DRAGOMIR, *Hermite-Hadamard's inequality and the p-HH-norm on the Cartesian product of two copies of a normed space*, Math. Inequal. Appl. **13**, 1 (2010), 1–32.

[41] U. S. KIRMACI, M. KLARIČIĆ BAKULA, M. E. ÖZDEMIR AND J. PEČARIĆ, *Hadamard-type inequalities for s-convex functions*, Appl. Math. Comput. **193** (2007), no. 1, 26–35.

[42] M. A. LATIF, *On some inequalities for h-convex functions*, Int. J. Math. Anal. (Ruse) **4** (2010), no. 29–32, 1473–1482.

[43] D. S. MITRINOVIĆ AND I. B. LACKOVIĆ, *Hermite and convexity*, Aequationes Math. **28** (1985), 229–232.

[44] D. S. MITRINOVIĆ AND J. E. PEČARIĆ, *Note on a class of functions of Godunova and Levin*, C. R. Math. Rep. Acad. Sci. Canada **12** (1990), no. 1, 33–36.

[45] C. E. M. PEARCE AND A. M. RUBINOV, *P-functions, quasi-convex functions, and Hadamard-type inequalities*, J. Math. Anal. Appl. **240** (1999), no. 1, 92–104.

[46] J. E. PEČARIĆ AND S. S. DRAGOMIR, *On an inequality of Godunova-Levin and some refinements of Jensen integral inequality*, Itinerant Seminar on Functional Equations, Approximation and Convexity (Cluj-Napoca, 1989), 263–268, Preprint, 89–6, Univ. "Babeş-Bolyai", Cluj-Napoca, 1989.

[47] J. PEČARIĆ AND S. S. DRAGOMIR, *A generalization of Hadamard's inequality for isotonic linear functionals*, Radovi Mat. (Sarajevo) **7** (1991), 103–107.

[48] M. RADULESCU, S. RADULESCU AND P. ALEXANDRESCU, *On the Godunova-Levin-Schur class of functions*, Math. Inequal. Appl. **12** (2009), no. 4, 853–862.

[49] M. Z. SARIKAYA, A. SAGLAM, AND H. YILDIRIM, *On some Hadamard-type inequalities for h-convex functions*, J. Math. Inequal. **2** (2008), no. 3, 335–341.

[50] E. SET, M. E. ÖZDEMIR AND M. Z. SARIKAYA, *New inequalities of Ostrowski's type for s-convex functions in the second sense with applications*, Facta Univ. Ser. Math. Inform. **27** (2012), no. 1, 67–82.

[51] M. Z. SARIKAYA, E. SET AND M. E. ÖZDEMIR, *On some new inequalities of Hadamard type involving h-convex functions*, Acta Math. Univ. Comenian. (N.S.) **79** (2010), no. 2, 265–272.

[52] M. TUNÇ, *Ostrowski-type inequalities via h-convex functions with applications to special means*, J. Inequal. Appl. **2013**, 2013:326.

[53] S. VAROŠANEC, *On h-convexity*, J. Math. Anal. Appl. **326** (2007), no. 1, 303–311.

Journal of Mathematical Inequalities
www.ele-math.com
jmi@ele-math.com

Journal of Mathematical Inequalities
www.ele-math.com
jmi@ele-math.com