

THE MOLECULAR DECOMPOSITION OF HERZ–MORREY–HARDY SPACES WITH VARIABLE EXPONENTS AND ITS APPLICATION

JINGSHI XU AND XIAODI YANG

Abstract. The molecular decomposition of Herz-Morrey-Hardy spaces with variable exponents is given. As its application, the boundedness of a convolution type singular integral on Herz-Morrey-Hardy spaces with variable exponents is obtained.

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