

THE FEKETE–SZEGŐ FUNCTIONAL PROBLEMS FOR SOME SUBCLASSES OF m -FOLD SYMMETRIC BI-UNIVALENT FUNCTIONS

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Abstract. In this paper, we introduce several new subclasses of the class of m -fold symmetric *bi-univalent* functions and obtain estimates of the Taylor-Maclaurin coefficients $|a_{m+1}|$, $|a_{2m+1}|$ and Fekete-Szegő functional problems for functions in these new subclasses. The results presented in this paper improve the earlier results of Ali *et al.* [1], Frasin and Aouf [6], and Srivastava *et al.* [14] in terms of the bounds as well as the ranges of the parameter under consideration. Our results also further generalize the results of Peng *et al.* [19].

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