

SOME VOLTERRA–FREDHOLM TYPE NONLINEAR INEQUALITIES INVOLVING FOUR ITERATED INFINITE INTEGRAL AND APPLICATION

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Abstract. In this paper, we establish some four iterated infinite integral inequalities, which includes a nonconstant term outside the integrals. The upper bound of the embedded unknown function is estimated explicitly by adopting novel analysis techniques, such as: change of variable, amplification method, differential and integration. The derived result can be applied in the study of qualitative properties of solutions of infinite integral equations.

Mathematics subject classification (2010): 26D15, 26D20, 45G10.

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