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CUBIC AND QUARTIC ρ–FUNCTIONAL INEQUALITIES IN FUZZY BANACH SPACES

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Abstract. In this paper, we solve the following cubic ρ-functional inequality

$$N(f(2x+y)+f(2x-y)-2f(x+y)-2f(x-y)-12f(x) \tag{0.1}$$
$$-\rho(4f(x+\frac{y}{2})+4f(x-\frac{y}{2})-f(x+y)-f(x-y)-6f(x)),t) \geqslant \frac{t}{t+\varphi(x,y)}$$

and the following quartic ρ-functional inequality

$$N(f(2x+y)+f(2x-y)-4f(x+y)-4f(x-y)-24f(x)+6f(y) \tag{0.2}$$
$$-\rho(8f(x+\frac{y}{2})+8f(x-\frac{y}{2})-2f(x+y)-2f(x-y)-12f(x)+3f(y)),t)$$
$$\geqslant \frac{t}{t+\varphi(x,y)}$$

in fuzzy normed spaces, where ρ is a fixed real number with $\rho \neq 2$.

Using the fixed point method, we prove the Hyers-Ulam stability of the cubic ρ-functional inequality (0.1) and the quartic ρ-functional inequality (0.2) in fuzzy Banach spaces.

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