

THE GEOMETRIC PROOF TO A SHARP VERSION OF BLUNDON'S INEQUALITIES

DORIN ANDRICA, CĂTĂLIN BARBU AND LAURIAN IOAN PIȘCORAN

Abstract. A geometric approach to the improvement of Blundon's inequalities given in [11] is presented. If $\phi = \min\{|A-B|, |B-C|, |C-A|\}$, then we proved the inequality $-\cos \phi \leq \cos \widehat{ION} \leq \cos \phi$, where O is the circumcenter, I is the incenter, and N is the Nagel point of triangle ABC . As a direct consequence, we obtain a sharp version to Gerretsen's inequalities [7].

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