

## BOUNDEDNESS FOR THE GENERALIZED COMMUTATOR OF SJÖLIN TYPE OPERATORS

XIAO YU AND SHANZHEN LU

*Abstract.* In this paper, we study the generalized commutators of Sjölin type operator  $T_{\alpha, A}^{a, m}$  defined by

$$T_{\alpha, A}^{a, m} f(x) = \int_{\mathbb{R}^n} K_{\alpha}^a(x-y) \frac{R_m(A; x, y)}{|x-y|^{m-1}} f(y) dy = \int_{\mathbb{R}^n} \frac{e^{i|x-y|^{\alpha}}}{|x-y|^{\alpha}} \frac{R_m(A; x, y)}{|x-y|^{m-1}} f(y) dy,$$

where  $R_m(A; x, y) = A(x) - \sum_{|\alpha| < m} \frac{1}{\alpha!} D^{\alpha} A(y) (x-y)^{\alpha}$  with  $m \in \mathbb{Z}^{+}$ .

By using the scale changing method, we prove that if  $D^{\gamma} A \in \dot{\Lambda}_{\beta}$  ( $0 < \beta < 1$ ) with  $|\gamma| = m-1$ ,  $m \geq 2$  or  $A \in \dot{\Lambda}_{\beta}$  ( $0 < \beta < 1$ ) when  $m=1$ ,  $T_{\alpha, A}^{a, m}$  is bounded on  $L^p(\mathbb{R}^n)$  for certain range of  $p$ .

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