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SOME REFINEMENTS OF OPERATOR INEQUALITIES FOR POSITIVE LINEAR MAPS

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Abstract. In this paper, we refine some operator inequalities as follows: Let A, B be positive operators on a Hilbert space with $0 < m \leqslant A \leqslant m' < M' \leqslant B \leqslant M$. Then for every positive unital linear map Φ and $p \geqslant 1$,

$$\Phi^p(A\nabla_t B)\Phi^p((A\sharp_t B)^{-1}) + \Phi^p((A\sharp_t B)^{-1})\Phi^p(A\nabla_t B) \leqslant \frac{(M+m)^{2p}}{2M^p m^p K^{\mu p}(h')},$$

and $p \geqslant 2$,

$$\Phi^{2p}(A\nabla B) \leqslant \left(\frac{K^2(h)(M^2+m^2)^2}{4^{\frac{2}{p}} K^{2\mu}(h') M^2 m^2}\right)^p \Phi^{2p}(H_t(A,B))$$

for all $t \in [0,1]$, where $\mu = \min\{t, 1-t\}$, $K(h) = \frac{(h+1)^2}{4h}$, $K(h') = \frac{(h'+1)^2}{4h'}$, $h = \frac{M}{m}$ and $h' = \frac{M'}{m'}$.

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