SOME REFINEMENTS OF OPERATOR
INEQUALITIES FOR POSITIVE LINEAR MAPS

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Abstract. In this paper, we refine some operator inequalities as follows: Let $A$, $B$ be positive operators on a Hilbert space with $0 < m \leq A \leq M < M' \leq B \leq M'$. Then for every positive unital linear map $\Phi$ and $p \geq 1$,

$$\Phi^p(A \triangledown B)\Phi^p((A \triangledown B)^{-1}) + \Phi^p((A \triangledown B)^{-1})\Phi^p(A \triangledown B) \leq \frac{(M+m)^2}{2M'm'K\mu(h')},$$

and $p \geq 2$,

$$\Phi^{2p}(A \triangledown B) \leq \left(\frac{K^2(h)(M^2+m^2)^2}{4K^2\mu(h')M^2m^2}\right)^p \Phi^{2p}(H_t(A,B))$$

for all $t \in [0,1]$, where $\mu = \min\{t, 1-t\}$, $K(h) = \frac{(h+1)^2}{4h}$, $K(h') = \frac{(h'+1)^2}{4h'}$, $h = \frac{M}{m}$ and $h' = \frac{M'}{m'}$.


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REFERENCES