

INEQUALITIES FOR GAUSSIAN HYPERGEOMETRIC FUNCTIONS

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Abstract. In this paper, we first shall present some inequalities for Gaussian hypergeometric functions, which generalize an identity involving the inverse hyperbolic tangent function. Further, the monotonicity of general hypergeometric function is proved. The obtained results of this paper improve some known results.

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