

SOME ESTIMATES OF HARDY OPERATORS AND THEIR COMMUTATORS ON MORREY–HERZ SPACES

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Abstract. In this paper, we study the boundedness of p -adic Hardy operators \mathcal{H}^p and $\mathcal{H}^{p,*}$ on the p -adic Morrey–Herz space. Furthermore, we establish the Lipschitz estimates for commutators of p -adic Hardy operators \mathcal{H}_b^p and $\mathcal{H}_b^{p,*}$ on the p -adic function spaces, such as the p -adic Lebesgue space, the p -adic Herz space and the p -adic Morrey–Herz space. Moreover, we also obtain the *CMO* estimates. All our results are also true for the fractional p -adic Hardy operator.

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