

## FUNCTIONAL INEQUALITIES IN MATRIX BANACH SPACES

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**Abstract.** Using the fixed point method, we prove the Hyers-Ulam stability of the following additive functional inequality and quadratic functional inequality

$$\begin{aligned} \|f(x+y) - f(x) - f(y)\| &\leq \left\| f\left(\frac{x+y}{2}\right) - \frac{1}{2}f(x) - \frac{1}{2}f(y) \right\|, \\ \|f(x+y) + f(x-y) - 2f(x) - 2f(y)\| \\ &\leq \left\| f\left(\frac{x+y}{2}\right) + f\left(\frac{x-y}{2}\right) - \frac{1}{2}f(x) - \frac{1}{2}f(y) \right\| \end{aligned}$$

in matrix Banach spaces, respectively.

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