SHARP BOUNDS FOR THE TOADER–QI MEAN IN TERMS OF HARMONIC AND GEOMETRIC MEANS

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Abstract. In the article, we present the greatest values $\alpha$ and $\lambda$, and the least values $\beta$ and $\mu$ in $[0,1/2]$ such that the double inequalities

$$H[\alpha a + (1 - \alpha)b, \alpha b + (1 - \alpha)a] < TQ(a, b) < H[\beta a + (1 - \beta)b, \beta b + (1 - \beta)a],$$

$$G[\lambda a + (1 - \lambda)b, \lambda b + (1 - \lambda)a] < TQ(a, b) < G[\mu a + (1 - \mu)b, \mu b + (1 - \mu)a]$$

hold for all $a, b > 0$ with $a \neq b$, where $H(a,b) = 2ab/(a+b)$, $G(a,b) = \sqrt{ab}$ and $TQ(a,b) = \frac{2}{\pi} \int_0^{\pi/2} a \cos^2 \theta b \sin^2 \theta d\theta$ are respectively the harmonic, geometric and Toader-Qi means of $a$ and $b$.


Keywords and phrases: Toader-Qi mean, harmonic mean, geometric mean, modified Bessel function.

REFERENCES