Journal of
Mathematical
Inequalities
Volume 11, Number 1 (2017), 121–127
doi:10.7153/jmi-11-11

SHARP BOUNDS FOR THE TOADER–QI MEAN IN TERMS OF HARMONIC AND GEOMETRIC MEANS

WEI-MAO QIAN, XIAO-HUI ZHANG AND YU-MING CHU

Abstract. In the article, we present the greatest values α and λ, and the least values β and μ in $[0,1/2]$ such that the double inequalities

$$H[\alpha a+(1-\alpha)b,\alpha b+(1-\alpha)a]<TQ(a,b)<H[\beta a+(1-\beta)b,\beta b+(1-\beta)a],$$

$$G[\lambda a+(1-\lambda)b,\lambda b+(1-\lambda)a]<TQ(a,b)<G[\mu a+(1-\mu)b,\mu b+(1-\mu)a]$$

hold for all $a,b>0$ with $a\neq b$, where $H(a,b)=2ab/(a+b)$, $G(a,b)=\sqrt{ab}$ and $TQ(a,b)=\frac{2}{\pi}\int_0^{\pi/2}a^{\cos^2\theta}b^{\sin^2\theta}d\theta$ are respectively the harmonic, geometric and Toader-Qi means of a and b.

Mathematics subject classification (2010): 26E60, 33C10.
Keywords and phrases: Toader-Qi mean, harmonic mean, geometric mean, modified Bessel function.

REFERENCES

[1] GH. TOADER, *Some mean values related to the arithmetic-geometric mean*, J. Math. Anal. Appl., 1998, **218** (2), 358–368.
[2] F. QI, X.-T. SHI, F.-F. LIU AND ZH.-H. YANG, *A double inequality for an integral mean in terms of the exponential and logarithmic means*, DOI: 10.13140/RG.2.1.2353.6800, available online at `http://www.researchgate.net/publication/278968439`.
[3] ZH.-H. YANG, *Some sharp inequalities for the Toader-Qi mean*, arXiv:1507.05430 [math.CA], available online at `http://lib-arxiv-008.serverfarm.cornell.edu/abs/1507.05430`.
[4] M. ABRAMOWITZ AND I. A. STEGUN, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, U. S. Government Printing Office, Washington, 1964.
[5] D. K. KAZARINOFF, *On Wallis' formula*, Edinburgh Math. Notes, 1956, **1956** (40), 19–21.
[6] M. BIERNACKI AND J. KRZYŻ, *On the monotonity of certain functionals in the theory of analytic functions*, Ann. Univ. Mariae Curie-Skłodowska, Sect. A, 1955, **9**, 135–147.
[7] ZH.-H. YANG, Y.-M. CHU AND M.-K. WANG, *Monotonicity criterion for the quotient of power series with applications*, J. Math. Anal. Appl., 2015, **428** (1): 587–604.
[8] G. PÓLYA AND G. SZEGŐ, *Problems and Theorems in Analysis I*, Springer-Verlag, Berlin, 1998.