A NOTE ON A WIENLANDT TYPE NORM INEQUALITY

XIAOHUI FU AND JUNJIAN YANG

Abstract. As a continuation of recent study on a Wielandt type norm inequality due to Lin [13, Conjecture 3.4], we prove the following result: Let $A \in M_n(\mathbb{C})$ satisfying $0 < m \leq A \leq M$, and let $X$ and $Y$ be $n \times k$ matrices such that $X^*X = Y^*Y = I_k$ and $X^*Y = 0$. Then for every 2-positive unital linear map $\Phi$, we have

$$\|\left(\Phi(X^*AY)\Phi(Y^*AY)^{-1}\Phi(Y^*AX)\right)^{\frac{p}{2}}\Phi(X^*AX)^{-\frac{p}{2}}\|$$

$$\leq \begin{cases} \frac{(M-m)^p}{M+m} \frac{(M^p + m^p)^{\frac{p}{2}}}{4M^p m^{\frac{p}{2}}} & 1 < p < 2 \\ \frac{(M-m)^p}{4M^p m^{\frac{p}{2}}} & p \geq 2 \end{cases}$$


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REFERENCES