

THE NATURAL APPROACH OF TRIGONOMETRIC INEQUALITIES — PADÉ APPROXIMANT

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Abstract. The aim of this work is to provide simple proofs of some remarkable trigonometric inequalities: Jordan inequality, Kober inequality, Becker-Stark inequality, Wu-Srivastava inequality. The proofs are based on Padé approximant method. We also obtain rational refinements for these inequalities.

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