

VOLUME EXTREMALS OF GENERAL L_p -CENTROID BODIES

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Abstract. Ludwig first discovered general L_p -centroid bodies in 2005 and Haberl and Schuster determined the extremals of their volume in 2009. In this paper, we extend the Haberl-Schuster result to all quermassintegrals and also obtain the extremals of the dual quermassintegrals of the polars of general L_p -centroid bodies.

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