

INEQUALITIES OF HERMITE–HADAMARD TYPE FOR FUNCTIONS OF SELFADJOINT OPERATORS AND MATRICES

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Abstract. Some inequalities of Hermite-Hadamard type for operator convex functions in Hilbert spaces are given. The case for matrices and convex functions is also considered. Examples for some particular functions of interest are provided as well.

Mathematics subject classification (2010): 26D15, 25D10.

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